**Ideal Gas**

**Prologue**

Now we want to investigate how we may determine the functional form of these potentials introduced previously. We will consider for simplicity that our substance is homogeneous and obeys the scaling relations introduced in the previous lecture. Then our potentials satisfy,



It will suffice, then, to determine the heat capacity CV(T,V) and equation of state p(T,V), to construct the potentials [these basically give us S and p for the free energy F]. For instance, forming the differential of U(T,V) we have:



What are these derivatives? Well,



and,



So we have:



A similar procedure for the entropy would yield:



What are these derivatives? Well,



and,



So we have:



With U(T,V) and S(T,V), we can get F(T,V) = U(T,V) – TS(T,V) for instance, which is one of the thermodynamic potentials. So from this we can construct everything we want. And of course we can invert these equations to construct other potentials as desired.

**Ideal gas**

Experimental measurements on dilute gases at high temperatures evinced the following formula for the heat capacity and pressure:



where f is the # of degrees of freedom of the gas molecule. What does this tell us about the energy, entropy, and chemical potential of the ideal gas for instance? Well filling into the formula we have:



So interestingly we see that an ideal gas’s internal energy has no dependence on volume. This could only be true if the gas’s energy was predominantly kinetic. And this also suggests that temperature is a measure of a gas’s kinetic energy. Well anyway, integrating both sides of the ∂U/∂T relationship we have:



Now let’s consider the entropy. Filling into the formula we have:



Integrating we get:



Let’s observe that in contradiction to the third law of thermodynamics, we don’t find that S → 0 when T → 0. But this is because the equation of state and heat capacity we’re using are only valid at high T. If we had formulas valid at low T as well, then our resulting formula for S would satisfy the third law. We can combine these to obtain the fundamental relationship S(U,V,N). We’ll solve for T in the U equation:



and filling in ….



We might be interested in calculating the other potentials as well. Consider H = U + pV. Filling in the known equation of state and all, we have:



And we could get F = U – TS for instance,



We could get μ as well. We might just solve for U(S,V,N) and then differentiate w/r to N. But is more customary to want μ in terms of T,p,N. To this end we will simply solve for G since G = μ. So then we’ll use G = F + pV



If we want (and we do) to put this in terms of pressure, rather than volume, then we can use p = kT/V to write:



The change in chemical potential, Δμ = ΔG/N, can be written as:



Another quantity of interest is the heat capacity at constant pressure. From the previous file, we have:



Finally, we can clean up a lot of our results for the potentials by defining some things. For instance, we can define U0 = 0, when T = T0, in which case we’ll have:



For S we can say:



and then define:



where Φ is a quantity intrinsic to the material itself. It could be determined by raising the temperature of the gas from T = 0, where S = 0, all the way up to T. And with these definitions we can write:



and we can also write:



Note we can express the temperature, pressure, and chemical potential of the gas in terms of S via:



For F we’d have:



and so:



which of course also simply follows from our U, and S expressions in terms of Φ. Chemical potential is:



Might observe that we can write this as:



Apropos the Gibbs energy we have:



and likewise for the chemical potential:



I’d kind of like to get the compressibility (isothermal). The isothermal compressibility is defined as:



So what is this?



**Potentials for mixture of gasses**

Say we have a N1 particles of gas type 1 and N2 particles of gas type 2. What are their potentials? I guess it is easiest to start with free energy? Then F = U – TS = U1 + U2 – T(S1 + S2) = F1 + F2. Energies are additive because there is no interaction between ideal gasses and entropies are addtive because the gasses are different. So,



Can get the chemical potnetials,



and entropy,



Of course internal energy is:



So we could invert this to put S in terms of its canonical variables U, V, N1, N2:



and could get others.